DESIGN LIFE DISTRIBUTIONS

RELIABILITY

Course Teacher

DESIGN LIFE ANALYSIS

- In life data analysis (also called "Weibull analysis"), the practitioner attempts to make predictions about the life of products.
- Design life analysis is carried out using population data by fitting a statistical distribution to life data from a representative sample of units.
- The parameterized distribution for the data set can then be used to estimate important life characteristics of the product such as reliability or probability of failure at a specific time, the mean life and the failure rate.
- The term life data analysis is used because actual field-based data is analyzed to make forecasts about your product’s life.

DESIGN LIFE ANALYSIS

- Population:
  - EVERY data point that has ever been or ever will be generated from a given characteristic
- Sample:
  - A portion (or subset) of the population, either at one time or over time
- Reliability prediction relies on life data (time-to-failure)
- Prediction accuracy is directly affected by the quality and completeness of the data used

DESIGN LIFE ANALYSIS

- Life data analysis requires the practitioner to:
    - Gather life data for the product.
    - Select a lifetime distribution that will fit the data and model the life of the product.
    - Estimate the parameters that will fit the distribution to the data.
    - Generate plots and results that estimate the life characteristics of the product, such as the reliability or mean life.
- **Attribute data** is discrete data such as yes-or-no variety. For example whether a light switch is turned on or off.
- **Variable data** is continuous having real numbers. It’s about measurement, such as set of values.

CENSORED DATA

- Data for which the exact event time is known is referred to as complete data.
- Censored data is any data for which we do not know the exact event time. There are three types of censored data; right censored, left censored, and interval censored.
- Tests may be terminated after a certain time (time-terminated) or after an certain number of failures (failure-terminated).
- Each of these types of test termination lead to a different type of censoring (type I and type II censoring).
- It is common to have a mixture of complete and censored data as most tests will have some failures and some survivors. It is not possible to perform an analysis with no failures and only right censored data.

CENSORED DATA

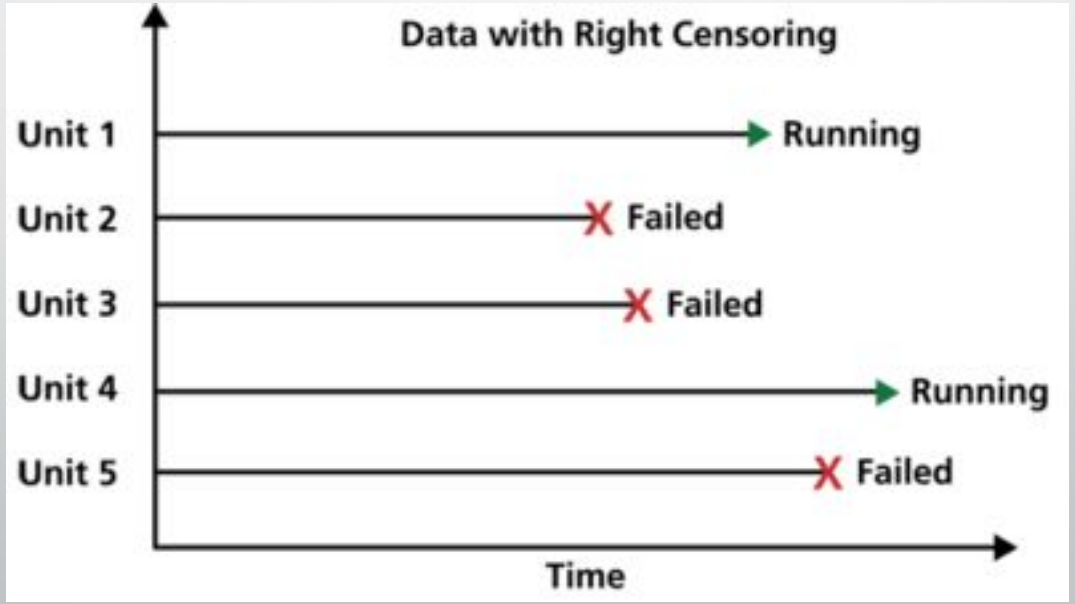
- In the context of reliability engineering we typically refer to events as "failures". Throughout reliability we will use "failures" to describe events.
- In other industries a range of terminology may be used to describe events.
- These often include "deaths" if studying living things such as in medical studies, or simply "events" if studying natural phenomena like flood events.
- It is common to refer to data as "times" but a variety of other units of measure may also be used such as cycles, rounds, etc.
- This depends on the type of data being collected and the units in which the life is measured.

COMPLETE DATA

Complete data is data for which we know the exact failure time. This is seen when all items under analysis have their exact failure times recorded. For example if we have 5 components under test, all of these components fail during the test, and the exact failure time is recorded then we have complete data.

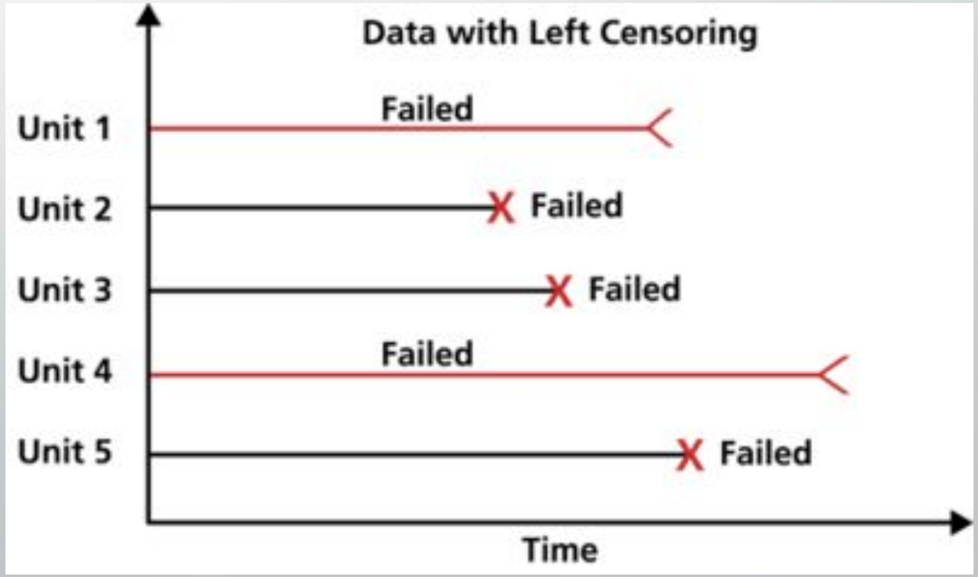
RIGHT CENSORED DATA

Right censored data is data for items that have not yet failed. They are considered "still alive" as their failure time has not yet occurred, though it is expected to occur at some point in the future. For example, consider a fatigue test with 5 components under test. The test is run for 100000 cycles and during this time 3 of the components fail. We record the 3 failure times and we also record the end of the test as the "right censored" time for the 2 un-failed components. Right censored data is also referred to as "suspensions" or "survivors".



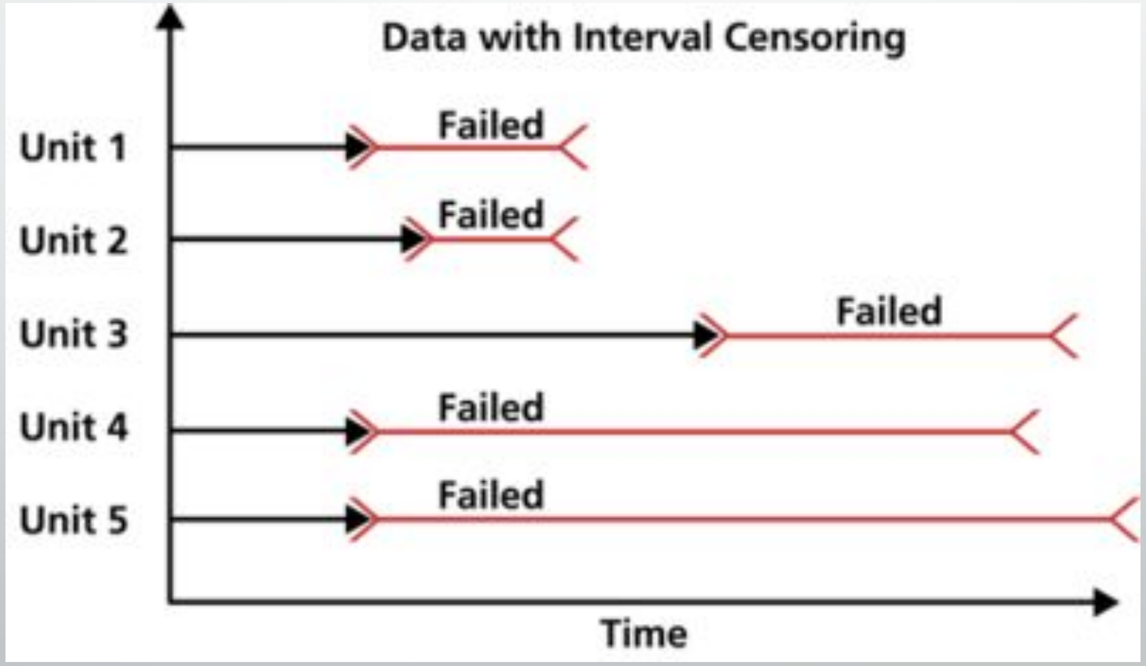
LEFT CENSORED DATA

Left censored data is data for items that failed before the start of the test. As an example of left-censoring, consider that some baboon troop always sleeps in the trees. We want to estimate at what time in the morning they descend from the trees, and let's assume that they do descend every day. We follow them for a number days, however, on some days they descend before we even arrive on the scene. If we arrive at 9 a.m. on day x and the baboons already descended, we have left-censored data. We want to know when they descended, but all we have is an upper limit (9 a.m.), because we know that at our time of arrival they had already descended. Analogously, we now know that the data point (time of descent on day x) is smaller than a certain value (9. a.m.).



INTERVAL CENSORED DATA

Interval censored data is when the exact failure time is not known but the lower and upper bounds of an interval surrounding the failure are known. For example, consider a failure that is discovered as part of an overhaul (deeper maintenance event). The exact failure time is not known but we do know that the item had not failed at the time of the last overhaul and it did fail before the current overhaul. We record the lower and upper times and treat this failure as interval censored data.



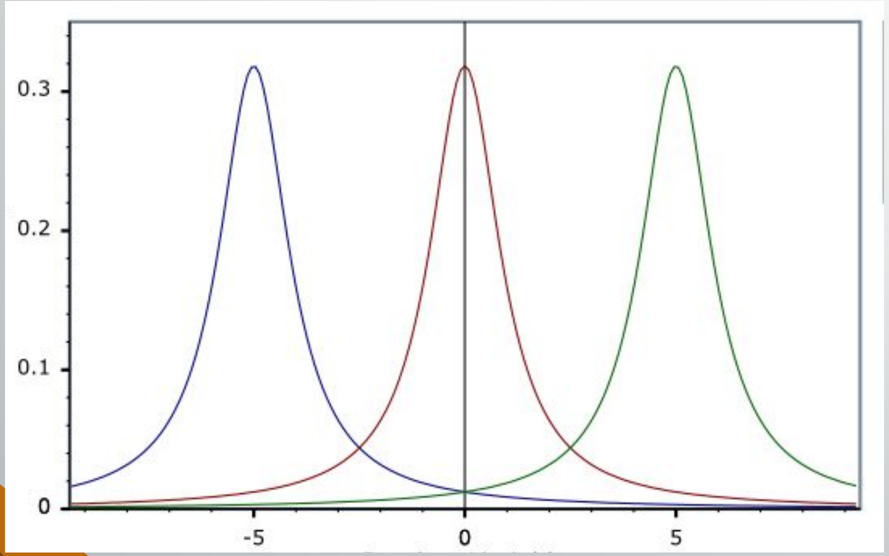
Life Distributions

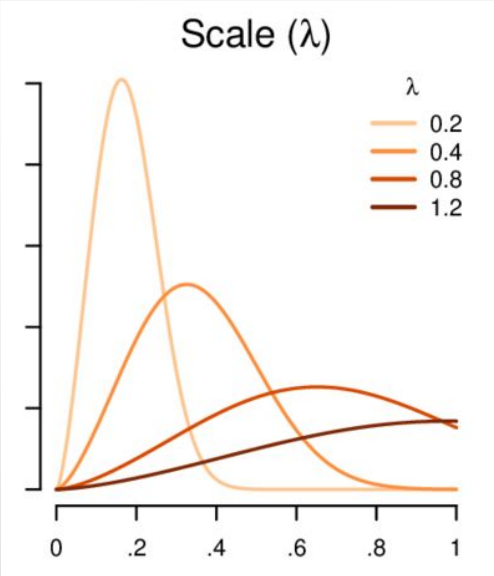
- Most commonly used distributions:
  - Exponential
    - Failure occurs due to purely random events
  - Weibull
    - Model material strength or times-to-failure of electronic and mechanical components, equipment or systems
  - Normal
    - Used for simple electronic and mechanical components
  - Lognormal
    - Used when failure modes are of a fatigue-stress nature

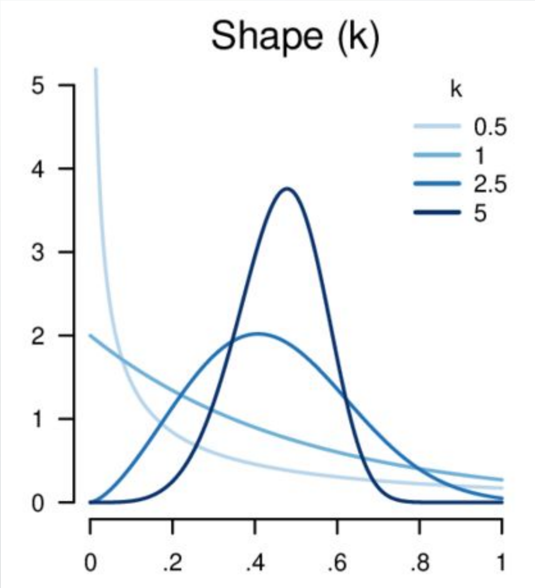
PARAMETER TYPES

- These distributions are parametric models or parametric families i.e. they have fixed number of parameters
- Typically, life distributions have a subset of the following three parameters

• Location	• Scale	• Shape







EXPONENTIAL DISTRIBUTION

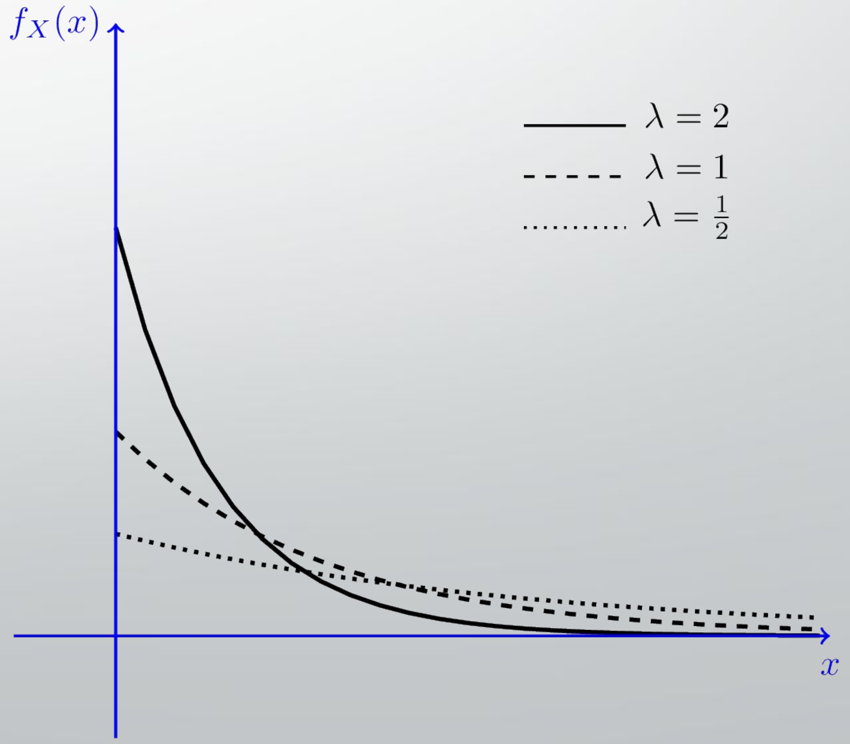
- The exponential distribution is one of the widely used continuous distributions. It is often used to model the time elapsed between events.
- A continuous random variable X is said to have an exponential distribution with parameter λ, if its PDF is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

$$Mean = \frac{1}{\lambda} \qquad Variance = \frac{1}{\lambda^2}$$

$$F(x) = 1 - e^{-\lambda x} \qquad x \geq 0$$

$$R(x) = e^{-\lambda x} \qquad x \geq 0$$



HAZARD FUNCTION FOR EXPONENTIAL DISTRIBUTION

- Hazard function for exponential distribution can be determine using following relationships:

$$h(x) = \frac{f(x)}{R(x)}$$

$$\Rightarrow \qquad h(x) = \frac{\lambda e^{-\lambda x}}{e^{-\lambda x}}$$

$$\Rightarrow \qquad h(x) = \lambda$$

- Above hazard function gives us CFR (constant failure rate)
- Exponential is the only distribution in which we observed hazard function to be of constant rate

Problem

Resistors has a constant failure rate of 0.04 failures per hour. What is the reliability at 25 hours? If 75 resistors were tested how many would be expected to failed after 25 hours?

Solution

Failure rate $(\lambda) = 0.04$ $\qquad R(T > 25) = ?$

$$\Rightarrow \quad R(T > 25) = e^{-0.04 \times 25} = 0.3679$$

Since $\quad F(x) = 1 - e^{-\lambda x}$

$$\Rightarrow \quad F(T > 25) = 1 - e^{-0.04 \times 25} \quad \Rightarrow \quad F(T > 25) = 0.6321$$

Out of 75 resistors, number of failed can be determine by multiplying 75 with the failure probability

$$\Rightarrow \quad Number\ of\ failed\ parts = 75 \times 0.6321 = 47.4$$

Weibull Distribution

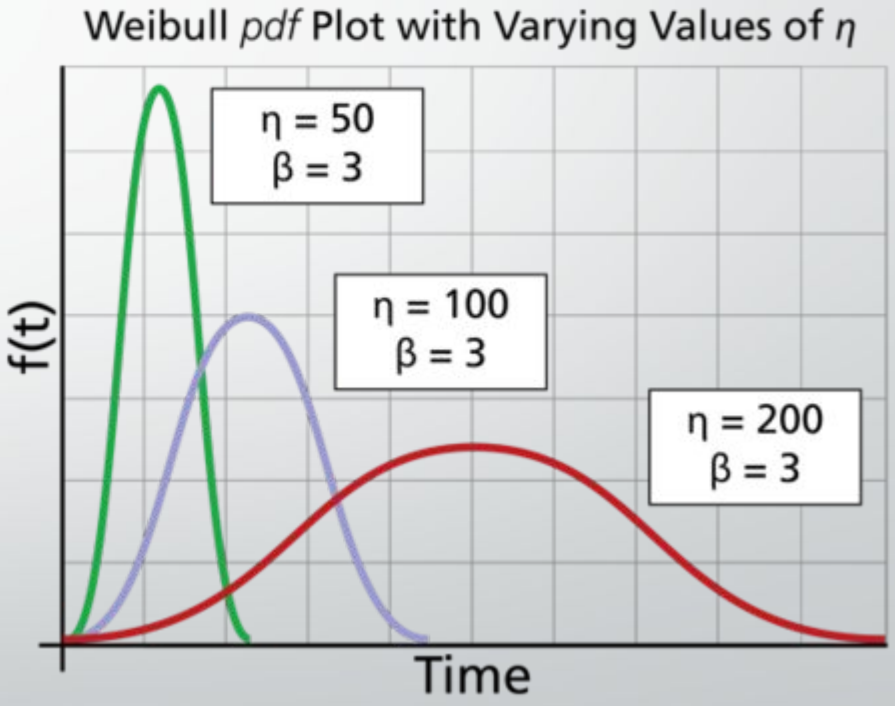
- Weibull distribution is one of the most widely used distributions in reliability
- Used to model:
  - Material strength
  - Times-to-failure of electronic and mechanical components, equipment or systems

- Two common versions of weibull distribution
  - 2-Parameter Weibull (β = shape parameter; η = scale parameter)
  - 3-Parameter Weibull (minimum life t_0 = location parameter)

Weibull Distribution

- Density function for two parameter Weibull

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad \eta > 0, \beta > 0, t \geq 0$$



$$Reliability\ function: \quad R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

$$Hazard\ function: \quad \lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

$$Behavior\ of\ \lambda(t) \rightarrow \begin{cases} 0 < \beta < 1 & DFR \\ \beta = 1 & CFR \\ \beta > 1 & IFR \end{cases}$$

Example

- Time to failure for a capacitor follows a weibull distribution with shape parameter 2 and scale parameter 300 months.

a) What is the reliability at 200 months?

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad \Rightarrow \quad R(t) = e^{-\left(\frac{200}{300}\right)^{2}} \quad \Rightarrow \quad R(t) = 0.6412$$

b) What is the probability of failure in 200 months?

$$F(t) = 1 - R(t) \quad \Rightarrow \quad F(t) = 0.3588$$

c) What is design life if the reliability of 0.9 is desired?

$$R(t) = 0.9 = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad \Rightarrow \quad t = 97.38\ months$$

- The wearout of a machine part has a Weibull distribution with $\beta = 3$ and $\eta = 300\ month$.
- What is the reliability at 200 month?

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \Rightarrow R(t) = e^{-\left(\frac{200}{300}\right)^{3}} \Rightarrow R(t) = 0.7436$$

- What is the design life for the desired reliability of 0.7435?

$$Design\ life = 200$$

- What is the reliability at 200 month if $\beta = 1$?

$$R(t) = e^{-\left(\frac{200}{300}\right)^{1}} \Rightarrow R(t) = 0.5134$$